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ECON 392
Written Assignment 1

The Prisoner's Dilemma is very useful at demonstrating the payoff relationships of negotiations. A buyer and a seller (who are the players of a negotiation game) have the option to negotiate on the price of a good, or to simply take the first offer that is presented to them. Like the Prisoner's Dilemma, both the buyer and the seller receive different payoffs depending on whether or not they chose to negotiate the price. The relationship of these payoffs correspond to the Prisoner's Dilemma, where *Doesn't Negotiate* corresponds to *Co-operates*, and *Negotiates* corresponds to *Defects*. If both the buyer and the seller do not negotiate, then the good is sold at the original price. If the buyer negotiates but the seller does not, the buyer could be able to negotiate a lower price, leaving the buyer very satisfied and the seller dissatisfied. If the seller negotiates but the buyer does not, then the seller could negotiate for a higher price, leaving the seller very satisfied and the buyer dissatisfied. If both the buyer and seller negotiate, then the good is sold at an agreed upon price, where both the buyer and seller are relatively satisfied. The utility relationship will therefore look like:

Buyer (Player 1): $u_B(N,D) > u_B(D,D) > u_B(N,N) > u_B(D,N)$

Seller (Player 2): $u_S(D,N) > u_S(D,D) > u_S(N,N) > u_S(N,D)$

where D and N are *Doesn't Negotiate* and *Negotiates* respectively.

I used to use a negotiating strategy when I used to play with Pokémon Trading Cards. While I was not much of a collector, I still needed to buy cards off of other people, which often involved negotiating the price of cards. While the cards have a price that the community generally accepts, I would try to negotiate for a price lower than the accepted value, and the seller understandably would try to negotiate for a price higher than the accepted value. Often what ends up happening is that we agree to exchange the card at the average value between their higher price and my lower price. The act of negotiating also comes at a cost of time and/or effort. While this measure does not have a direct monetary value, we assign it one to use it when calculating payoffs. To model this game, let us define the following variables:

$p \rightarrow$ the accepted price of a card

$c \rightarrow$ the "cost" to negotiate

$v_1 \rightarrow$ the price I value the card to be

$v_2 \rightarrow$ the price the seller values the card to be

Using these values, we have the following payoffs:

$u_X(D,D)$

- In the case that no one negotiates, the card is exchanged at the accepted price p. When I buy the card, I receive a good that I value at v_1 and I pay price p for it. Therefore, my payoff is $u_B(D,D) = v_1 - p$. When the seller sells the card, they lose a good they value at v_2, and they receive price p for it. Therefore, their payoff is $u_S(D,D) = p - v_2$.

$u_X(N,D)$

- In the case that I negotiate but the seller does not, the card is exchanged at the price I value the card at: v_1. When I buy the card, I receive a good that I value at v_1 and I pay price v_1 for it. I also incur the effort cost of negotiating. Therefore, my payoff is $u_B(N,D) = v_1 - v_1 - c = -c$. When the seller sells the card, they lose a good they value at v_2, and receive price v_1 for it. Therefore, their payoff is $u_S(N,D) = v_1 - v_2$.

$u_X(D,N)$

- In the case that the seller negotiates but I do not, the card is exchanged at the price they value the card at: v_2. When I buy the card, I receive a good that I value at v_1 and I pay price v_2 for it. Therefore, my payoff is $u_B(D,N) = v_1 - v_2$. When the seller sells the card, they lose a good they value at v_2, and receive price v_2 for it. They also incur the effort cost of negotiating. Therefore, their payoff is $u_S(D,N) = v_2 - v_2 - c = -c$.

$u_X(N,N)$

- In the case that both the seller and I negotiate, the card is exchanged at the average value of what the seller and I think the card is worth: $\frac{v_1+v_2}{2}$. When I buy the card, I receive a good that I value at v_1 and I pay price $\frac{v_1+v_2}{2}$ for it. I also incur the effort cost of negotiating. Therefore, my payoff is $u_B(N,N) = v_1 - \frac{v_1+v_2}{2} - c = \frac{v_1-v_2}{2} - c$. When the seller sells the card, they lose a good they value at v_2, and receive price $\frac{v_1+v_2}{2}$ for it. They also incur the effort cost of negotiating. Therefore, their payoff is $u_S(N,N) = \frac{v_1+v_2}{2} - v_2 - c = \frac{v_1-v_2}{2} - c$.

The payoff matrix looks like the following:

	Doesn't Negotiate	Negotiates
Doesn't Negotiate	$v_1 - p,\ p - v_2$	$v_1 - v_2,\ -c$
Negotiates	$-c,\ v_1 - v_2$	$\frac{v_1-v_2}{2} - c,\ \frac{v_1-v_2}{2} - c$

If we use arbitrary values for p, c, v_1, and v_2, we will most likely not get a utility relationship that defines the Prisoner's Dilemma. We therefore use the utility relationships to further restrict the variables, and only allow for variables that satisfy the utility relationships. To further analyze how each variable should be related to one another, we will substitute these payoffs into the utility relationships and interpret the results.

Case 1: $u_B(N,D) > u_B(D,D)$ and $u_S(D,N) > u_S(D,D)$

Rearranging the inequalities:

$$-c > v_1 - p \qquad \text{and} \qquad -c > p - v_2$$
$$p - v_1 > c \qquad\qquad\qquad v_2 - p > c$$

These inequalities tell us that the difference between the accepted price of the card and the amount that the seller and I value the card needs to be greater than the cost of negotiating. If the differences were less than the cost of negotiating, then the seller and I would rather exchange the card at its accepted value, rather than negotiate for what we think the card is worth.

Case 2: $u_B(N,D) > u_B(N,N)$ and $u_S(D,N) > u_S(N,N)$

Rearranging the inequalities:

$$- c > \frac{v_1 - v_2}{2} - c \quad \text{and} \quad - c > \frac{v_1 - v_2}{2} - c$$
$$v_2 > v_1 \qquad\qquad v_2 > v_1$$

These inequalities tell us that the amount that the seller values the card is greater than the amount that I value the card. This is intuitive, since the seller is trying to increase the value to receive more money from the sale, while I am trying to decrease the value to pay less money into the sale.

Case 3: $u_B(N,D) > u_B(D,N)$ and $u_S(D,N) > u_S(N,D)$

Rearranging the inequalities:

$$- c > v_1 - v_2 \quad \text{and (or)} \quad - c > v_1 - v_2$$
$$v_2 - c > v_1 \qquad\qquad v_2 - v_1 > c$$

These inequalities tell us that the difference between the amount the seller values the card and the amount I value the card needs to be greater than the cost of negotiating. This is because if the difference were less than the cost of negotiating, then the seller would rather sell the card at my value instead of negotiating, and vice-versa.

Case 4: $u_B(D,D) > u_B(N,N)$ and $u_S(D,D) > u_S(N,N)$

Rearranging the inequalities:

$$v_1 - p > \frac{v_1 - v_2}{2} - c \quad \text{and} \quad p - v_2 > \frac{v_1 - v_2}{2} - c$$
$$\frac{v_1 + v_2}{2} > p - c \qquad\qquad \frac{v_1 + v_2}{2} < p + c$$

These inequalities describe the upper and lower bound of what the average value between my value of the card and the seller's value of the card should be. It cannot be any more than the accepted price of the card plus the negotiating cost, and it cannot be less than the accepted price of the card minus the negotiating cost.

Case 5: $u_B(D,D) > u_B(D,N)$ and $u_S(D,D) > u_S(N,D)$

Rearranging the inequalities:

$$v_1 - p > v_1 - v_2 \quad \text{and} \quad p - v_2 > v_1 - v_2$$
$$v_2 > p \qquad\qquad p > v_1$$

These inequalities state that the seller's value of the card must be higher than the accepted price of the card, while my value of the card must be less than the accepted price of the card. Like case 2 this is intuitive, as the seller is trying to increase the value to receive more money from the sale, while I am trying to decrease the value to pay less money into the sale.

Case 6: $u_B(N,N) > u_B(D,N)$ and $u_S(N,N) > u_S(N,D)$

Rearranging the inequalities:

$$\frac{v_1-v_2}{2} - c > v_1 - v_2 \qquad \text{and} \qquad \frac{v_1-v_2}{2} - c > v_1 - v_2$$
$$\frac{v_2-v_1}{2} > c \qquad\qquad\qquad\qquad \frac{v_2-v_1}{2} > c$$

While these inequalities have no physical meaning on their own, we see that they can also be derived by adding and rearranging the inequalities in case 1, and thus justify the same behaviour as case 1 does.

Lastly, we will analyze what happens when we use these values to solve for a mixing strategy equilibrium. Since the Prisoner's Dilemma models a strictly dominant strategy, this equilibrium should not exist. To see if our variables reflect this, we will attempt to solve for the mixed strategy equilibrium.

Suppose I play a mixing strategy $\alpha_B = (q,\ 1-q)$. The seller would be indifferent between not negotiating and negotiating when their expected utilities for each play are equal. Therefore:

$$EU_S(\alpha_B, D) = q(p - v_2) + (1-q)(-c)$$

$$EU_S(\alpha_B, N) = q(v_1 - v_2) + (1-q)(\tfrac{v_1-v_2}{2} - c)$$

$$q(p - v_2) + (1-q)(-c) = q(v_1 - v_2) + (1-q)(\tfrac{v_1-v_2}{2} - c)$$

$$qp - qv_2 - c + qc = qv_1 - qv_2 + \tfrac{v_1-v_2}{2} - c - q\tfrac{v_1-v_2}{2} + qc$$

$$q(p - v_1 + \tfrac{v_1-v_2}{2}) = \tfrac{v_1-v_2}{2} \;\rightarrow\; q(\tfrac{2p-2v_1+v_1-v_2}{2}) = \tfrac{v_1-v_2}{2} \;\rightarrow\; q = \tfrac{v_1-v_2}{2p-v_1-v_2}$$

By symmetry, the same result occurs when solving for the seller's equilibrium.

At first glance, it appears that we have a solution for the mixed strategy equilibrium. However, we have inequalities that tell us more about the numerator and denominator of q. We know by definition that $q > 0$. In case 2, we see that $v_2 > v_1 \rightarrow 0 > v_1 - v_2$, and therefore the numerator is negative. In case 1, if we subtract the two inequalities from each other, we get:

$$p - v_1 > c \ , \ v_2 - p > c$$

$$p - v_1 - v_2 + p > c - c \ \rightarrow \ 2p - v_1 - v_2 > 0$$

Therefore, the denominator is positive. Since the numerator is negative and the denominator is positive, q evaluates to a negative number. Since negative probabilities are impossible, there exists no mixed strategy equilibrium, which is consistent with the Prisoner's Dilemma.

Upon equating the payoffs of each outcome to their corresponding utility relationship, we see that the utility relationships justify the behaviour of each player, as well as show a behavioural justification as to why the strategy ceases to model the Prisoner's Dilemma when certain inequalities are not met. These utility relationships also justify why the negotiation strategy is strictly dominant. These are the reasons why the Prisoner's Dilemma model negotiation strategies very well.

While I used this model to describe my behaviours in trading cards, it can effectively be used to model most negotiation strategies in general. Understanding the relationship between p, c, v_1, and v_2 is crucial in understanding one's best outcome, since these values can also identify whether or not the "game" is worth "playing" to begin with. While I cannot claim to have been formally aware of this strategy when I would negotiate, I was still behaving as the strategy predicted. I knew I had to always negotiate, I knew to expect my "opponent" to always negotiate, and I knew that I would rather buy the card at the seller's value when negotiating was too much of a burden. For future negotiations, I will recall the analysis that I did for this assignment to ensure that I am playing the best strategy possible.